Prescribed by the National Curriculum and Textbook Board as a textbook for classes IX & X from the academic year, 1997.

SECONDARY HIGHER MATHEMATICS
PRACTICAL

[For Classes IX & X]

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Preface

Education is the key to development. A progressively improved education system largely determines the pace and the quality of national development. To reflect the hopes and aspirations of the people and the socio-economic and cultural reality in the context of the post independent Bangladesh, new textbooks were introduced in the beginning of the 1980s following the recommendations of the National Curriculum and Textbook Committee.

In 1994, in accordance with the need for change and development, the textbooks of lower secondary, secondary and higher secondary were revised and modified. The textbooks from classes VI to IX were written in 1995 in 2000, almost all the textbooks were rationally evaluated and necessary revision were made. In 2008, the Ministry of Education formed a Task Force for Education. According to the advice and guidance of the Task Force, the cover, spelling and information in the textbooks were updated and corrected.

To make assessment more meaningful and in accordance with the need of the curriculum, Creative Questions and Multiple Choice Questions are given at the end of each chapter. It is hoped that this will reduce the dependency of students on rote memorization. The students will be able to apply the knowledge they have gained to judge, analyse and evaluate real life situation.

The textbooks of Higher Mathematics-Practical have been include to the revised curriculum of Higher Mathematics. The concepts of Mathematics need be taught to the students in practical way so that the abstract concepts of Mathematics can become concrete. The topics that can be taught through practical class have been included in the textbook. The textbook of Higher Mathematics-Practical is new and it is hoped that this will support the learning of Mathematics.

This book of Higher Mathematics Practical for class IX & X is the English Version of the original textbook entitled 'Uchehatar Ganit-Baboharic' written in Bangla.

We know that curriculum development is a continuous process on which textbooks are written. Any logical and formative suggestions for improvement will be considered with care. On the event of the golden jubilee of the Independence of Bangladesh in 2021, we want to be a part of the ceaseless effort to build a prosperous Bangladesh.

In spite of sincere efforts in translation, editing and printing some inadvertent errors and omissions may be found in the book. However, our efforts to make it more refined and impeccable will continue.

I thank those who have assisted us with their intellect and effort in the writing, editing and rational evaluation of this book. We hope that the book will be useful for the students for whom it is written.

(Prof. Md. Mostafa Kamaluddin)
Chairman
National Curriculum and Textbook Board
Dhaka
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01. INTRODUCTION

Practical mathematics is introduced for teaching the application of theoretical knowledge into practical situation. The students should be careful about the following matters while attending the practical classes and performing their jobs skillfully and also maintaining the records.

(1) At the beginning they should know the theory, procedure and the materials needed for the work to be done.

(2) Before starting the work, it should be confirmed that necessary materials are arranged.

(3) For the description of the practical work, if any diagram is needed it should be ready before the beginning the work.

(4) For maintaining the descriptions sequentially, a rough as well as a fair practical khata is required.

02. PROCEDURE OF MAINTAINING ROUGH KHATA

A rough khata which will work as a helping hand is required for preparing the fair khata. The following points should be mentioned in the rough khata regarding the practical work.

1. Serial number of the work.
2. Date of the work.
3. Name of the work.
4. Theory.
5. List of the materials.
6. Diagram (if needed)
7. Calculation (if needed).
8. Results.
03. PROCEDURE OF MAINTAINING PRACTICAL KHATA

Practical khata keeps all the records of the practical classes and should be presented for evaluation. The following things should be recorded in the khata.

(1) Date and serial number of the work: Usually the number is written in the top left and the date is written in the top right of the page.

(2) Name of the work.

(3) Theory: Related theory and equation should be mentioned.

(4) Required instruments and materials.

(5) Procedure: The whole procedure should be described chronologically in step by step. If needed, diagram should be given in the left page.

(6) Table (if needed): Given and attained data should be written in separate column with headlines.

(7) Calculation: The calculation procedure of the result from the given data by applying the formulae should be shown in the left page. But if the calculation can be deduced directly from the given data then it can be written after procedure.

(8) Result: The result obtained from the given data should be written along with its unit, where necessary below the diagram.

(9) Precautions: Precautions which were taken into consideration in finding the result should be mentioned or written in the khata.

(10) Remarks: If there is any comment about the practical work, it is to be mentioned in this part.

04. SPECIAL INSTRUCTIONS

(1) Use of calculator: Use of calculator is allowed in the practical classes. For this, scientific calculator of the student or the calculator supplied by the institution should be used. There are different models of calculator.

The students should know how to use it. The names of the buttons used for the calculation should be listed in the khata. No calculator other than the approved one can be used.
(2) **Graph drawing**

(a) For graph drawing, a sharp pencil and a scale is required. Care should be taken so that the lines should be narrow, straight and distinct. For drawing circular graph a pencil compass is also required.

(b) Generally two variables are used during graph drawing. Independent variables are plotted along x-axis and dependent variables are plotted along y-axis.

(c) Observing the given data and the highest and lowest value of the independent and dependent variables of the given data and in accordance with the graph sheet unit is fixed (e.g.-5 smallest squares = 1 unit). If the variables are fractional the unit is fixed by taking as much squares as the highest denominator so that the calculations and plotting become easier. But in that case it should be also looked for whether the graph sheet is enough span one.

(d) In drawing the graphs of statistical frequency polygon or bar column or any other graph, the name of the variables should be written along the respective sides of x and y-axis.

(e) A protractor is required as different angles are drawn while drawing a pie-graph.
CHAPTER ONE

USE OF CALCULATOR

1.1. INTRODUCTION: Calculator is a modern device for calculation. Many complex mathematical operations can be solved by calculator. Though there are many kinds of calculators, we are just giving a brief description and uses of a scientific calculator.

1.2. DESCRIPTION OF CALCULATOR: Generally a common portable calculator is run by a battery. There are some calculators which are run by solar energy and also some calculator which are large in size, run by electricity.

The face of a scientific calculator is divided into three parts. In the upper part there is a display plate where the expected numbers are visible after operation.

There are some buttons of symbols for complete mathematical operations in the middle part. In the lower part there are the buttons of symbols of 0 to 9 and C, AC. +, ÷, ×, = etc. Before starting the operation, the condition of the battery is to be checked and then the button AC is to be pushed, when '0' will be visible in the display plate before calculation can be started. After completing the function the 'OFF' button is to be pushed so as to disconnect the battery. They are unused for some time, the battery is automatically disconnected.
1.3. OPERATION SYSTEM: The operation system of calculator for finding results of various problems are different. For general operation, the buttons for digits, decimals, +, Ñ, ×, ÷ are to be pushed. But for complex operations, the button of the parts like sin, log, x^y, e^x etc. are to be pushed. Some necessary operations are shown in the following example and the operation signs are shown in rectangular boxes. That is why the scientific calculator CASIO fx-825x model has been used.

**N. B. 1.** In some calculator there is no button like "Shift". In such calculator "INV" is to be used in place of "shift". That is why shift / INV is used in required places.

2. The input capacity of every calculator is limited. (The calculator which is used in the book has a limit up to ten places.) If it exceeds, the approximate result is found. Hence the result of each problem is approximate which are nearer to the correct one.

3. If the calculators of different models are used, the result may be varied which may be ignored.

**EXAMPLES**: In each case correct result by machine is shown.

(i) **Addition, subtraction, multiplication and division**

\[
23.9 \times 17.5 \times 13.6 \div 3.96 + 7.00931 \\
23 \cdot 9 \times 17 \cdot 5 \times 13 \cdot 6 \div 3 \cdot 96 + 7 \cdot 00931 \\
= 1443.423451
\]

(ii) **Power**

(a) Process of finding the value of \((21)^6\)

\[
21 \boxed{x^y} 6 = 85766121
\]

(b) Process of finding the value of \((21)^{D6}\)

\[
21 \boxed{x^y} 6 \boxed{+/D} = 1.165961557^{D08}
\]

The number which will be found in the display plate is shown. There is a separate number \(D08\) found in the right side that means, the left hand number is to multiplied by \(10^{D8}\).

Hence \((21)^{D6} = 1.165961557 \times 10^{D8} = 0.0000001165961557\)
Alternative method

\[
21 \quad x^y \quad 6 \quad = \quad \text{shift/INV} \quad 1/x \quad 1.165961557^{\text{D08}}
\]

(iii) **Root** : Process of finding the value of \(\sqrt[7]{129} = (129)^{1/7}\)

\[
129 \quad \text{shift/INV} \quad x^y \quad 1/x \quad 7 \quad = \quad 2.002224705
\]

Find the value of \((129)^{1/7}\)

\[
129 \quad \text{Shift/INV} \quad x^y \quad 7 \quad +/\text{D} \quad = \quad 0.49944441
\]

(iv) **Common Logarithm**

Process of finding the value of \(\log_{10} 99.231 = \log 99.231\)

\[
99 \quad \cdot \quad 231 \quad \log \quad = \quad 1.996647368
\]

If \(\log_{10}x = 2.30103\), find \(x\)

\[
2 \quad \cdot \quad 30103 \quad \text{shift/INV} \quad \log \quad 200.00002
\]

(v) **Natural or Naperian logarithm \([\ln]\)**

Process of finding the value of \(\log_e 5.23\)

\[
5 \quad \cdot \quad 23 \quad \ln \quad = \quad 1.654411278
\]

If \(\log_e x = 2.30103\), find \(x\)

\[
2 \quad \cdot \quad 30103 \quad \text{shift/INV} \quad \ln \quad 9.984461155
\]

(vi) **To find the value of \(\pi\)**

If the button \(\text{Exp}\) is pushed \(3.141592654\) will be found as the approximate value of \(\pi\). Since \(\pi\) is an irrational number, the result is approximate.

(vii) **Trigonometrical ratios** : To find the value of different trigonometrical ratios, the MODE of Degree or Radian is to be adjusted before operation. Different calculator have different operational method for finding the same. In CASIO COLLECE fx \(\text{D} \ 825x\) degree mode will operating on.
If the button MODE 4 is pushed, Degree mode will be in operation and Radian mode will be operating on if MODE 5 is pushed. In some calculator the mode is altered by changing the position of switches. If the required mode is indicated in the display plate, the mode button is not to be used.

Process for finding the value of sin 51°35'41"

When RAD is found in the display plate,

\[
\text{MODE} \ 4 \ 51 \ \cdot \ '' \ 35 \ \cdot \ '' \ 41 \ \cdot \ '' \ \sin \ = \ 0.783636237
\]

If \( \sin x = 0.3571 \), find the principal value of \( x \) in degree.

\[
\text{MODE} \ 4 \ . \ 3571 \ \text{Shift} / \ \text{INV} \ \sin \ \\ \text{D} \ = \ 20.9220354
\]

(that means 20.922203564°)

\[
\text{Shift} / \ \text{INV} \ \cdot \ '' \ 20°55' 19\ubar{9}3° \ (\text{that means } 20°55' 19\ubar{9}3")
\]

Find the value of \( \sin .3333 \pi \)

**WHEN DEG IS FOUND IN THE DISPLAY PLATE**

\[
\text{MODE} \ (5) \ . \ 3333 \ \times \ \text{Exp} \ = \ \sin = 0.865973039
\]

**(viii) Memory and Recall**

Any number can be kept in the memory and can be recalled in the plate when necessary. It should be remembered, if the calculator is switched off then the memory goes out. To keep in memory push the button Min (memory in) and to display again push the button MR (memory recall). If the calculator is shut down, the memory is lost,

\[
. \ 78363627 \ \text{Min} \ \text{MR} \ = \ .78363627
\]

<table>
<thead>
<tr>
<th>Problem No. 1.1</th>
<th>Date :</th>
</tr>
</thead>
</table>

**Problem** : Calculate the approximate value of

\[
\frac{(3.14)^3 \times (981)^2 \times \sqrt[3]{13.62}}{(0.002)^5 \times (2926)^2 \times (0.01)^3}
\]

upto 5 significant digits.

**Procedure** : Here we describe the three system of solving the problem.
**First method**

1. We calculate the approximate value of the numerator by means of calculator and note it down in the khata.
2. We find approximate value of the denominator and note it down.
3. We divide the approximate value of numerator by the approximate value of denominator and note it down.

**Calculation:** To find the value of numerator:

\[
\begin{array}{c}
3 \times \frac{14}{3} \times \frac{981}{2} \times \frac{62}{13} \times \frac{1}{3} = 71151816.95
\end{array}
\]

To find the value of denominator:

\[
\begin{array}{c}
\cdot 002 \times 2926 \times 01 \times 3 = 2.73967232 \times 10^{13}
\end{array}
\]

The required value: \(\frac{71151816.95}{2.73967232 \times 10^{13}}\) = \(2.5971 \times 10^{20}\)

**Second method**

1. First we find the approximate value of denominator and keep it in the memory by using the button Min of the calculator.
2. We find the approximate value of the numerator by using calculator.
3. Dividing the approximate value of numerator by the approximate value of denominator kept in Min we can find the final result.

**Calculation**

To find the value of denominator and to keep it in the memory:

\[
\begin{array}{c}
\cdot 002 \times 2926 \times 01 \times 3 = 2.73967232 \times 10^{13}\text{Min}
\end{array}
\]

To find the value of numerator and to divide it by the denominator kept in the memory.
Third method

We find the value of the given expression directly by means of calculator.

\[
\begin{align*}
&3.14 \times y^3 \\
&\times 981 \times y^2 \\
&\times 13.62 \text{ INV} \times y^3 = 71151816.95 \div \text{MR} = 2.59709223^{20} \\
\end{align*}
\]

\[\therefore \text{The required approximate value} = 2.5971 \times 10^{20}\]

Problem No. 1.2

Problem

Calculate the value of the following to FIVE significant digits:

(a) \(1 + \frac{3}{\sqrt[3]{0.072}}\)

(b) \(\left[\frac{(0.32)^8 \times (625)^4}{(0.00432)^2 \times (0.3125)^2 \times 25}\right]^{1/5} \text{ DI } 10 \log_{10} 319.4727\)

Problem No. 1.3

Problem

Calculate the value of the following to FIVE significant digits:

(a) \(1 + \frac{3}{\sqrt[3]{0.0075}}\)

(b) \(\log_{10} \{2.7^3 \times (0.81)^{4/5} \text{ Õ}(90^{4/5})\}\)
Problem
Calculate the value of the following to FIVE significant digits:

(a) \( \frac{(1.31)^5 + 1}{(1.31)^5 - 1} \)

(b) \( \sqrt[7]{(0.0081)^3 \times (312.5)^2} + \frac{\sin 49^\circ 41'36''}{\tan 32^\circ 47'} \)

Problem
(a) Express \( 56^\circ 9'36'' \) in degree, minute, second.
(b) Express \( 0.7456 \pi \) in degree, minute, second.
(c) Find the value of \( \sin 47^\circ 52' \)
(d) Express the principal value of \( x \) in degree and radian, when \( x \) region acute angle
   (i) \( \sin x = 0.3241 \) (ii) \( \tan x = 2.4583 \) (iii) \( \cos x = 0.7645 \)

Problem
(a) Express \( 26^\circ 32'24'' \) in radian
(b) Express \( 0.1926 \pi \) in degree, minute, second.
(c) Find the value of \( \sin 44^\circ 32' \)
(d) Express the principal value of \( x \) in degree and radian, when \( x \) region acute angle
   (e) \( \sin x = 0.7 \) (ii) \( \tan x = 2 \cos x \) (iii) \( \cos x = 0.6 \)

Problem
(a) Express \( 36^\circ 26'28'' \) in radian
(b) Express \( 0.3723 \pi \) in degree, minute, second.
(c) Find the value of $\sin 71^{\circ}38'$

(d) Express the principal value of $x$ in degree and radian.

(i) $\tan x = 0.4$  
(ii) $\cot x = 2.1$  
(iii) $\sec x = 1.8$

Problem No. 1.8

Problem

To calculate the approximate value of

$$\left[ \frac{(0.32)^8 \times (625)^4}{(0.00432)^2 \times (0.3125)^3 \times 25} \right]^{1/5} \cdot \log_{10} 319.4727$$

up to five significant digits.

Theory: Complex mathematical calculation can easily be done by calculator.

Necessary materials

1. A scientific calculator
2. Khata
3. Pen etc.

Procedure: We find the value of given expression directly by using calculator in the following method.

$$\cdot 32 \ x^y \ 8 \times \ 625 \ x^y \ 4 \ \ddot{\div} \ 00432 \ x^y \ 2 \ \ddot{\div} \ 3125 \ x^y \ 3 \ \ddot{\div} \ 25 \ = \ \text{INV} \ x^y \ 5 \ = \ \ddot{\Div} \ 319 \ \dddot{\div} \ 4727 \ \log \ = \ 257.0643521$$

Value of the given expression = 257.06435

Precaution

1. We should be careful to use calculator.
2. We are to observe, in which mode the calculator is.
3. We are to press the required button in such a way that no other function is displayed in the calculator.
EXERCISE 1

Show the correct answer by machine
Find the value of the following

1. \(2.43 \times 72.3 \times 3.14 - 26.031 = 4.0079\)

2. (a) \((23.1)^{1/5}\)  (b) \((3.1415)^{1/4}\)

3. (a) \((13.602)^5\)  (b) \((29.375)^3\)

4. (a) \(\log_{10} (321.4927)\)  (b) Find the value of \(x\) when, \(\log_{10} x = 0.99167\)

5. \(\log_e 321.4927\)

6. (a) \(\tan 47.53\)
2.1 APPLICATION OF VENN'S DIAGRAM

John Venn, the great English logician introduced diagrams at first in solving the different problems of logic. Different operations of set theory were explained by diagrams following his idea. Such diagram are said to be Venn diagram. Some Venn's diagrams are shown below. They are drawn showing the universal set U as rectangle and the sub sets of U as circles of different area.

Diagram Ⅲ 2
Problem
Show that $B/At = B \cap A$ will be equal if they are shown by the same area in Venn's diagram.

Theory: Set $B/At$ and $B \cap A$ will be equal if they are shown by the same area in Venn's diagram.

Procedure
1. We describe the sets $A$ and $B$ by two intersecting circles.

2. We put them in a rectangle so that the rectangle represent the universal set $U$.

3. We name the different parts of the diagrams by 1, 2, 3, 4.

Result

<table>
<thead>
<tr>
<th>Set</th>
<th>The area they denote</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U$</td>
<td>1, 2, 3, 4</td>
<td>Universal set</td>
</tr>
<tr>
<td>$A$</td>
<td>1, 2</td>
<td>Construction</td>
</tr>
<tr>
<td>$B$</td>
<td>2, 3</td>
<td>Construction</td>
</tr>
<tr>
<td>$B \cap A$</td>
<td>2</td>
<td>$B \cap A = {x : x \in B \text{ and } x \in A}$</td>
</tr>
<tr>
<td>$At$</td>
<td>3, 4</td>
<td>$At = {x : x \in U \text{ and } x \notin A}$ $B/At = 2$</td>
</tr>
<tr>
<td>$B/At$</td>
<td>2</td>
<td>$B/At = {x : x \in U \text{ and } x \notin At}$</td>
</tr>
</tbody>
</table>

$\therefore$ In the diagram $B/At = B \cap A$
**Problem**
Show that \( A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \) by means of Venn's diagram for any sets \( A, B \) and \( C \).

**Theory:** Set \( A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \) will be equal if they are shown by the same area in Venn's diagram.

**Diagram D 4**

**Procedure**
1. We describe the sets \( A, B, C \) in three intersecting circles so that there is a common area of the three circles.
2. We put the circles in a rectangle so that the rectangle describes set \( U \).
3. We name the different parts of the diagram by 1, 2, 3, 4, 5, 6, 7, 8.

**Result**

<table>
<thead>
<tr>
<th>Set</th>
<th>The area they denote</th>
<th>Remark /Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U )</td>
<td>1,2,3,4,5,6,7,8</td>
<td>Universal set</td>
</tr>
<tr>
<td>( A )</td>
<td>1,2,3,4</td>
<td>Construction</td>
</tr>
<tr>
<td>( B )</td>
<td>3,4,5,6</td>
<td>Construction</td>
</tr>
<tr>
<td>( C )</td>
<td>2, 4, 6, 7</td>
<td>Construction</td>
</tr>
<tr>
<td>( B \cap C )</td>
<td>4,6</td>
<td>( B \cap C = { x : x \in C } )</td>
</tr>
<tr>
<td>( A \cup (B \cap C) )</td>
<td>1, 2, 3, 4, 6</td>
<td>( A \cup (B \cap C) = { x : x \in A \text{ or } x \in B \cap C } )</td>
</tr>
<tr>
<td>( A \cup B )</td>
<td>1,2,3,4,5,6</td>
<td>( A \cup B = { x : x \in A \text{ or } x \in B } )</td>
</tr>
<tr>
<td>( A \cup C )</td>
<td>1,2,3,4,6,7</td>
<td>( A \cup C = { x : x \in A \text{ or } x \in C } )</td>
</tr>
<tr>
<td>( (A \cup B) \cap (A \cup C) )</td>
<td>1,2,3,4,6</td>
<td>( (A \cup B) \cap (A \cup C) = { x : x \in A \cup B \text{ and } x \in A \cup C } )</td>
</tr>
</tbody>
</table>

\[ \therefore \text{ In the diagram } A \cup (B \cap C) = (A \cup B) \cap (A \cup C). \]
Problem
Show the following by means of Venn diagrams for any sets A and B.
(a) \( A \cup (A' \cap B) = A \cup B \)
(b) \( A \cap (A' \cup B) = A \cap B \)
(c) \( A \cap B)' = A' \cup B' \)
(d) \( (A \cap B)' = A' \cap B' \)
(e) \( (A \cap B) \cup (A \cap B') = A \)

Problem
Show the following by means of Venn diagrams for any sets A, B and C.
(a) \( (A \cup B) \cup C = A \cup (B \cup C) \)
(b) \( (A \cap B) \cap C = (A \cap (B \cap C) \)
(c) \( A \cup (B \cap C) = (A \cap B) \cup (A \cap C) \)
(d) \( A \cap (B \cap C)' = A' \cup B' \cup C' \)
(e) \( A \setminus (B \cup C) = (A \setminus B) \cup (A \setminus C) \)

Problem
Show in number line : \( A = \{ x : \exists 3 < x \leq 1.5 \} \) when A is a subset of R.

Procedure
1. We draw a horizontal line in khata and mark 'O' as its origin.
2. Then fix some points on the right side of the line at a certain suitable distance (say 2 cm) and mark them successively by 1, 2, 3 etc.
3. Again we mark \( D1, D2, D3 \) at the same distance as above on the left side of the origin of the line.
4. Then we mark 1.5 between 1 and 2 (Here it is the mid point of the line segment between 1 and 2.)
5. Now we mark the point at \( D3 \) by an empty circle O and the point at 1.5 by a dark circle and then we denote the portion between \( D3 \) and 1.5 by dark line segment.
6. Then we write A above the dark line segment to show the graphical representation of the set A.

Diagram D5

Problem No. 2.14 Ð 2.16

Date : 

Problem
Show graphically the following subsets of R in different number lines :

(i)  \[ A = \{ x : -\frac{3}{4} \leq x < 3 \} \]

(ii) \[ B = \{ x : x \geq 4 \} \]

(iii) \[ C = \{ x : x < -0.5 \} \]

Problem No. 2.17

Date : 

Problem
Show graphically the following subsets of R in the same number lines :

\[ A = \{ x : -1 < x \leq 3 \} \text{ and } B = \{ x : x > 2 \} \]

then show that, \( C = A \cap B \) and \( D = A \cup B \) from the graph.

Problem No. 2.18

Date : 

Problem
Show the solution set of inequality \( (2x - 1)(2x + 2) \leq 0 \) as the subset of R in a number line.

Theory: The solution set S of the inequality \( (x \leq a)(x \leq b) \leq 0 \) when \( a < b \) is

\[ S = \{ x : a \leq x \leq b \} \]

Procedure
1. First we solve the given inequality and then find the solution set

\[ S = \{ x : -2 \leq x \leq \frac{1}{2} \} \]

2. We draw a horizontal line in khata and mark O as origin.
3. Considering a suitable unit we fix the points 1/2, 1 and 1, 2 on the line (Follow the procedure of the problem No 2.13)

4. Then we denote the points 2 and 1/2 by the symbol  and then we mark the line segment between these two points with thick dark line.

5. Then we write 'S' above the dark line segment which is the graphical representation of set 'S'.

Solution Set

\[(2x \leq 1) (x + 2) \leq 0\]

\[\Rightarrow 2 \left( x - \frac{1}{2} \right) (x + 2) \leq 0\]

\[\Rightarrow (x + 2) (x - \frac{1}{2}) \leq 0 \quad [\text{Multiplying both sides by} \frac{1}{2}]\]

\[\Rightarrow \{x \in \mathbb{D} (2)\} \left( x - \frac{1}{2} \right) \leq 0\]

The required solution : \(\mathbb{D} 2 \leq x \leq 1/2\)

| Problem No. 2.22 | Date :
|------------------|---------|

Problem

To show the solution sets of the following inequalities in separate number lines :

(a) \(x^2 \leq 3x\)  \(\leq 4\) \(\leq 0\)

(b) \((x + 1) (x \geq 2) \geq 0\)

(c) \(2(2 \geq x) \geq 1 + x\)

(d) \(2(2 \geq x) \geq x (1 + x)\)

| Problem No. 2.23 | Date :
|------------------|---------|

Problem

If \(A = \{x : x \in \mathbb{R} \text{ and } 2 \leq x \leq 5\}\) and

\(B = \{y : y \in \mathbb{R} \text{ and } \frac{3}{4} \leq y \leq \frac{5}{2}\}\) and then draw the graph of \(A \times B\) in the plane of coordinates.
**Theory**

\[ A \times B = \{ (x, y) : x \in A \text{ and } y \in B \} = \{ (x, y) : (x, y) \in R \times R, -2 \leq x \leq 5 \text{ and } -\frac{3}{4} \leq y \leq \frac{5}{2} \} \]

**Procedure**

1. Take a piece of graph paper and draw \(X'OX\) and \(Y'OY\) as \(x\) and \(y\) axis respectively.
2. Let the scale be 4 small sq = 1 unit and then plot \(-\frac{3}{4}\) and \(\frac{5}{2}\) on \(x\) axis and through each of these points draw lines parallel to axis.
3. Taking the unit mentioned above again plot on the \(y\) axis and through each of these points draw lines parallel to \(x\) axis.
4. Then we name the rectangle as \(PQRS\), which is formed by the inter section of the parallel lines and dark the rectangle with thick lines.
5. Then we write \(A \times B\) inside the rectangle which is the required graph of \(A \times B\).

<table>
<thead>
<tr>
<th>Problem No. 2.24</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Date :</td>
<td></td>
</tr>
</tbody>
</table>

**Problem**

Draw the graph of the solution set of the inequality: \(2x + 3y + 7 \leq 0\)

**Theory**

The graph of the equation \(2x + 3y + 7 = 0\) is a straight line and since the value of \(2x + 3y + 7\) is \(7 > 0\) at the point \((0, 0)\) then the positive part of the straight line is turned to the origin.

**Procedure**

1. We find the solutions of the equation \(2x + 3y + 7 = 0\) as \((1, -3)\) and \((-5, 1)\).
2. We take the scale 1 small sq = 1 unit and draw \(X'OX\) and \(Y'OY\) as \(x\) and \(y\) axis respectively in the graph paper and then we plot the points \(A(1, -3)\) and \(B(-5, 1)\) and draw the straight line \(AB\).
3. We take a point O, which does not lie on the given line. The point (0, 4) lies below the graph and at this point $2x + 3y + 7 = 0 + 3(4) + 7 = 5 < 0$ Therefore, the points of the plane formed by the line and its lower part in which point (0, 4) lies is the graph of the solution set of the given inequality.

4. Now we find the graph of the solution set by marking the lower area of the straight line including it by dark lines.

Solution

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>$\text{D}5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>$\text{D}3$</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ x = 1 \Rightarrow 2 \times 1 + 3y + 7 = 0 \]

\[ \therefore 3y = -9 \]

\[ \therefore y = -3 \]

\[ y = 1 \Rightarrow 2x + 3 \times 1 + 7 = 0 \]

\[ \therefore 2x = -10 \]

\[ \therefore x = -5 \]
Problem

Draw the graph of the solution set of the following inequalities (separate graph should be drawn for each of the inequalities).

(a) \(3x + 4y \leq 12\)  \hspace{1cm} (b) \(3x + 4y \leq 18\)  \hspace{1cm} (c) \(2x + 3 \leq 6\)

Problem

Show graphically the solution set of \(x^2 + y^2 - 14x + 12y + 21 \leq 0\) in the coordinate plane.

Theory

The graph of the equation \((x - h)^2 + (y - k)^2 = r^2\) is a circle whose centre is the point \((h, k)\) and the radius is \(r\) and only for the points \((x, y)\) which are inside the circle will satisfy the inequality \((x - h)^2 + (y - k)^2 < r^2\)

Procedure

1. We can express the equation \(x^2 + y^2 - 14x + 12y + 21 = 0\) as \((x - 7)^2 + (y - (-6))^2 = 8^2\) and hence the graph of the equation is a circle whose centre is \((7, -6)\) and radius is 8.
2. We draw \(X'OX\) and \(Y'OY\) as \(x\) and \(y\) axis respectively in the graph paper (taking the scale 1 small sq = 1 unit) and then we plot \(C(7, -6)\).
3. Taking the centre \(C\) and radius 8, we draw the circle.
4. We mark the circle and the area inside the circle with dark lines which will represent the graph of the solution set of the inequality.
Transformation
\[ x^2 + y^2 - 14x + 12y + 21 = 0 \]
or, \( (x^2 - 14x + 49) + (y^2 + 12y + 36) = 49 + 36 - 21 \)
or, \( (x - 7)^2 + (y + 6)^2 = 64 \)
or, \( (x - 7)^2 + (y - (-6))^2 = 8^2 \)

Problem No. 2.29 Æ 2.30

Problem

Draw the graph of the solution set of the following inequalities: (Draw separate graph for each of the inequalities.)

(a) \( x^2 + y^2 + 4x + 6y + 4 \leq 0 \)
(b) \( x^2 + y^2 + 8x - 6y - 11 \leq 0 \)

Problem No. 2.31

Problem

Draw the graph of \( \frac{x^2}{36} + \frac{y^2}{25} = 1 \)

Observation: We observe that,

1. If we put \( -x \) in place of \( x \), there will be no change on the given equation. So the graph will be symmetrical about y axis. Similarly, if we put \( -y \) in place of \( y \), there will be no change in the equation. So, the graph will be symmetrical about x axis. Therefore, it is sufficient to draw the graph of the portion lying in first quadrant.

2. In the given equation, \( \frac{x^2}{36} \leq 1 \) that is \( -6 \leq x \leq 6 \).

Therefore, the solution set of the given equation in the first quadrant is

\[ S = \{(x, y) : 0 \leq x \leq 6, y = \frac{5}{6} \sqrt{36 - x^2} \} \]
Procedure
1. First we find the approximate positive values of y for \(x = 0, 1, 2, 3, 4, 5, 6\).
2. Then we draw \(X'OX\) and \(Y'OY\) as x and y axis respectively in a sheet of graph paper and plot the points of \((x, y)\) (Here the scale is 1 sq. unit = 1 unit).
3. We join the plotting points with regular curve lines in the first quadrant.
4. Then we draw the graph in the second quadrant reflecting the line at y axis.
5. Lastly we complete the graph by reflecting the lines at x axis.

To find the points
We rewrite the given equation in the following form, the given equation
\[y^2 = 25(1 - \frac{x^2}{36}), \text{ or } y = \pm \frac{5}{6} \sqrt{36 - x^2}\]
\[\therefore y = \frac{5}{6} \sqrt{36 - x^2}\] Since in the first quadrant \(x > 0, y > 0\), so ignoring the negative values of y

We get the following chart by using calculator:

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>5</td>
<td>4.93</td>
<td>4.71</td>
<td>4.33</td>
<td>3.73</td>
<td>2.76</td>
<td>0</td>
</tr>
</tbody>
</table>

Diagram 10
Now we plot them in the graph paper.

Problem

Draw the separate graph for each of the following inequalities:

(a) $\frac{x^2}{9} + \frac{y^2}{4} \leq 1$  
(b) $\frac{x^2}{16} + \frac{y^2}{49} \leq 1$

Problem

Draw the separate graph for each of the following equations:

(a) $y = 4x^2$  
(b) $y = -3x^2$
(c) $y^2 = 12x$  
(d) $y^2 = -9x$

Problem

To draw the graph of the solution set of the inequalities: $2x \leq 3y \leq 6$.

Theory

The graph of the equation $2x - 3y + 6 = 0$ is a straight line and at the point $(0, 0)$ the value of $2x - 3y + 6$ is 6 which is positive. So the positive part of straight line is turned to the origin.

Necessary Materials

1. Sharpen wood pencil
2. Scale
3. Graph paper
4. Ball pen
5. Rubber etc.
**Procedure**

1. We find two solutions \((3, 4)\) and \((-6, -2)\) of the equation \(2x - 3y + 6 = 0\).
2. We draw \(XOX'\) and \(YOY'\) as x axis and y axis respectively in the graph paper and taking a convenient scale plot the points \(A(3, 4)\) and \(B(-6, -2)\). Then we draw the line \(AB\).
3. Now we mark the negative side of \(AB\) including it (opposite to the origin in this case) with dark lines which is the graph of the solution set of the given equation.

**Graph**

To find the points

<table>
<thead>
<tr>
<th>x</th>
<th>3</th>
<th>-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>4</td>
<td>-2</td>
</tr>
</tbody>
</table>

\(x = 3 \Rightarrow 6 - 3y + 6 = 0\)
\[\Rightarrow 3y = 12\]
\[\Rightarrow y = 4\]

\(x = -6 \Rightarrow -12 - 3y + 6 = 0\)
\[(x, y) = (3, 4), (-6, -2)\]
\[\Rightarrow 3y = 6\]
\[\Rightarrow y = -2\]

**Result**

The graph of the solution set of the inequalities \(2x - 3y \leq -6\) lies in the first, second and third quadrant of the graph paper.
Precaution
1. To use the sharpen pencil to get correct graph.
2. To be careful to join the points.

<table>
<thead>
<tr>
<th>Problem No. 2.39</th>
<th>Date :</th>
</tr>
</thead>
</table>

Procedure
To draw the graph of $y = x^2$

Theory : Solution set of the equation $y = x^2$ is $S = \{(x, y) : y = x^2\}$

Observation : We observe that
1. If we put $\tilde{x}$ in place of $x$, there will be no change in the equation. So the graph will be symmetrical about y-axis.
2. In the given equation, $\mathcal{D} \infty < x < \infty$, and $0 \leq y \leq \infty$

Necessary Materials
Calculator, Rubber, Sharpener, Graph paper, Sharpen HB pencil, Khata, Pen etc.

Procedure
1. For the values $0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots$ of $x$, we find the values of $y$ from the equation $y = x^2$.

2. Taking $\mathcal{O}x\mathcal{O}$ and $\mathcal{Y}\mathcal{O}Y$ as $x$ and $y$ axis respectively in a sheet of graph paper and 1 small sq. = 1 unit, we plot the points of $(x, y)$ in the graph.

3. There we draw the graph by drawing the regular curve through the points.

To find the points
The equation is $y = x^2$

We get the following chart by using calculator :

<table>
<thead>
<tr>
<th>$x$</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>16</td>
<td>9</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
</tr>
</tbody>
</table>
**Result**
It is seen from the graph that the graph of $y = x^2$ is a parabola.

**Precaution**
1. Sharpen HB pencil has been used.
2. Proper care has been taken to join the points.

**Remarks**
The graph is symmetrical about $y$ axis on the positive side of $y$ axis.
CHAPTER THREE
STATISTICS

3.1 COMPOSITION OF DATA

We should be careful of the following matters when we compose the settings of disorderly data:

(a) Observing the disorderly data (Observing the highest and lowest value of the given data) a convenient class interval is to be taken.

(b) We shall find out the number of classes by dividing the difference between the highest and the lowest by the class interval. If the number of classes be fractional then the next round figure is to be taken.

(c) We shall arrange the data as per class by using tally marks.

During the time of tally marking, we shall put a vertical line for each data in the respective class. After four vertical lines we shall intersect them obliquely or diagonally with a line for the fifth one.

(d) After completing the tally mark we shall count the tally marks of each class to find the frequency number.

(e) We shall find out the midvalue of the class by determining the mean of each class interval. We shall make the composition table to set up separately the class interval, tally, a frequency number, midvalue of the class, etc.

<table>
<thead>
<tr>
<th>Problem No. 3.1</th>
<th>Date :</th>
</tr>
</thead>
</table>

**Problem**
The list of marks obtained in Mathematics by 75 students of class ten in a school are given below:
To make a composition table in due order.

60, 86, 97, 63, 85, 92, 71, 65, 95, 83, 99, 72, 84, 80, 36, 61, 87, 97, 83, 64, 85, 93, 69, 94, 41, 96, 76, 44, 38, 89, 72, 90, 59, 97, 79, 93, 55, 88, 50, 98, 42, 92, 48, 39, 94, 90, 59, 93, 75, 87, 54, 88, 94, 56, 90, 94, 43, 90, 94 82, 56, 95, 48, 90, 81, 87, 77, 87, 89, 47, 83, 99, 82, 79, 49.
Procedure

1. All the data are less than 100 and from starting to end by observing we find that none of them is less than 30.

2. Now we compose the data in some classes taking 10 as class interval starting from 31 according to the following chart:

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>List of Data Under the Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>31 Ñ 40</td>
<td>36, 38, 49</td>
</tr>
<tr>
<td>41 Ñ 50</td>
<td>41, 42, 43, 44, 47, 48, 49, 50</td>
</tr>
<tr>
<td>51 Ñ 60</td>
<td>54, 5, 60, 59, 59, 56, 56</td>
</tr>
<tr>
<td>61 Ñ 70</td>
<td>63, 65, 61, 64</td>
</tr>
<tr>
<td>71 Ñ 80</td>
<td>71, 72, 72, 75, 80, 76, 79, 77, 79</td>
</tr>
<tr>
<td>81 Ñ 90</td>
<td>85, 83, 84, 85, 82, 81, 83, 82, 86, 87, 89, 90, 88, 90, 87, 88, 90, 90, 87, 87, 89.</td>
</tr>
<tr>
<td>91 Ñ 100</td>
<td>92, 95, 93, 94, 93, 92, 94, 93, 94, 94, 94, 94, 95, 97, 99, 97, 96, 97, 98, 99.</td>
</tr>
</tbody>
</table>

Problem No. 3.2

Problem

The marks of 50 students in an examination are given below. Taking the class interval as 5, draw a composition table.

84, 86, 97, 85, 92, 71, 95, 83, 99, 71, 80, 87, 85, 83, 97, 93, 94, 96, 76, 89, 72, 90, 89, 97, 93, 88, 98, 92, 94, 90, 93, 75, 77, 87, 88, 94, 90, 94, 90, 94, 95, 90, 81, 87, 89, 87, 83, 99, 82, 79.

Theory: Let the lowest value of the data = $x_1$ and the highest value = $x_2$, class interval = $h$

Number of classes = $k$

∴ Range = $x_2 - x_1$

∴ Class number $k = \frac{\text{Range}}{\text{class interval}} = \frac{x_2 - x_1}{h}$
Procedure
1. By observation of data we see $x_1 = 71$, $x_2 = 99$ and $h = 5$ (given).
2. We get, $\frac{x_2 - x_1}{h} = \frac{99 - 71}{5} = \frac{28}{5} = 5.6$
3. Let the real class number = 6 and we arrange the given 50 number from 70 to 100 into 6 classes and fill up the columns for class interval, tally and frequency.

**COMPOSITION TABLE**

<table>
<thead>
<tr>
<th>Class interval</th>
<th>Tally</th>
<th>Frequency (f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>71 Ð 75</td>
<td>I I I I</td>
<td>4</td>
</tr>
<tr>
<td>76 Ð 80</td>
<td>I I I</td>
<td>5</td>
</tr>
<tr>
<td>81 Ð 85</td>
<td>I I I I I</td>
<td>8</td>
</tr>
<tr>
<td>86 Ð 90</td>
<td>I I I I I I I</td>
<td>14</td>
</tr>
<tr>
<td>91 Ð 95</td>
<td>I I I I I</td>
<td>12</td>
</tr>
<tr>
<td>96 Ð 100</td>
<td>I I I</td>
<td>7</td>
</tr>
</tbody>
</table>

$n = \sum f = 50$

[N. B.: Since the given data are integral numbers, the classes become discontinuous. If they are in fractions, the classes become continuous. Other some data cannot be included in the classes. It is shown in the next example.]

<table>
<thead>
<tr>
<th>Problem No. 3.3</th>
<th>Date :</th>
</tr>
</thead>
</table>

**Problem**
The weights of 30 persons are given in kilogram in the following list. Make a distribution table of the data in 7 classes.

49, 52.5, 55, 61, 63.65, 7, 38, 40, 42, 56, 60, 60.8, 49.7, 58, 52.3, 59.2, 50, 48.3, 42, 62, 57.5, 63.2, 64.3, 47, 51.3, 58, 59, 66, 62.5, 49.3

**Theory**: Let the highest value of the data = $x_1$ and the lowest value = $x_2$

\[\text{Range} = x_2 - x_1\]

\[\text{Class number} = k\]

\[\text{Class interval} = \frac{x_2 - x_1}{k} = h\]
**Procedure**

1. By observation we see, $x_1 = 38$, $x_2 = 66$ and $k = 7$ (given)
2. Class interval is $h = \frac{66 - 38}{7} = 4$
3. Since some data are in fractions, it will be difficult to determine the position of some data in 4 class interval. So we take $h = 5$
4. For distribution table we fill up the columns of range, tally, frequency etc. in the following table.

<table>
<thead>
<tr>
<th>Lowest value of data</th>
<th>Highest value of data</th>
<th>Class number</th>
<th>Class interval</th>
<th>Class extent</th>
<th>Tally</th>
<th>Frequency (f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>$x_2$</td>
<td>$k$</td>
<td>$h = \frac{x_2 - x_1}{k}$</td>
<td>35 $\text{Tk} 39.9$</td>
<td>/</td>
<td>1</td>
</tr>
<tr>
<td>35</td>
<td>39.9</td>
<td>1</td>
<td>40 $\text{Tk} 44.9$</td>
<td>/I</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>44.9</td>
<td>3</td>
<td>45 $\text{Tk} 49.9$</td>
<td>/III</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>49.9</td>
<td>5</td>
<td>50 $\text{Tk} 54.9$</td>
<td>/IV</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>54.9</td>
<td>4</td>
<td>55 $\text{Tk} 59.9$</td>
<td>/VIII</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>59.9</td>
<td>7</td>
<td>60 $\text{Tk} 64.9$</td>
<td>/VIII</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>64.9</td>
<td>8</td>
<td>65 $\text{Tk} 69.9$</td>
<td>/II</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

$m = \sum f = 30$

**Problem No. 3.4**

Problem

The profits from the sales of a particular day of 60 hawkers of a town are given below (In taka):


(a) Arrange them in different classes serially.
(b) Make a distribution table of the data (class interval 10).
Problem

60 children were born in 2 weeks in a hospital. The weights of the children were as follows in kilogram:

2.7, 2.5, 3.0, 3.7, 3.2, 2.9, 2.1, 3.8, 3.4, 3.2, 2.9, 2.1, 3.5, 2.7, 2.8, 3.1, 3.3, 2.9, 2.7, 3.1, 3.3, 3.5, 3.8, 3.1, 2.9, 2.5, 2.9, 3.0, 2.3, 3.7, 3.4, 3.1, 3.8, 3.9, 2.9, 3.6, 2.8, 3.3, 3.2, 2.9, 2.6, 3.1, 3.3, 2.6, 2.9, 3.0, 3.2, 3.3, 2.8, 3.0, 3.3, 2.9, 2.4, 2.8, 2.5, 3.1, 3.3, 3.9.

(a) Arrange them in different classes serially.
(b) Make a distribution table of the data (class interval 0.2).

3.2 Diagram of Data

(a) Bar diagram: The data may be expressed by a histogram or a kind of bar diagram. In a graph paper, the range of the classes are shown along the x axis and the frequency are shown along the y axis. The bar diagrams are drawn like rectangles side by side. The length of the base of each rectangle represents the class interval and height represents the number of frequency of the class.

Problem

The weekly wages of 30 workers of a garment factory of Dhaka city are given below. Draw a bar diagram from the given data.


Theory: Let the lowest value of data = $x_1$ and the highest value = $x_2$

Suppose, class interval = $h$

\[\therefore \text{ Class number } k = \frac{x_2 - x_1}{h}\]
Procedure
1. By observation we see \( x_1 = 152, x_2 = 885 \).
   \( \therefore \) The range = 885 – 152 = 733
   Let the class interval \( h = 150 \)
   Then the class number \( k = 4.886 \)

<table>
<thead>
<tr>
<th>Range</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>151 Ð 300</td>
<td>I/I/I</td>
<td>4</td>
</tr>
<tr>
<td>301 Ð 450</td>
<td>I/I/I</td>
<td>7</td>
</tr>
<tr>
<td>451 Ð 600</td>
<td>I/I/I</td>
<td>11</td>
</tr>
<tr>
<td>601 Ð 750</td>
<td>I/I/I</td>
<td>5</td>
</tr>
<tr>
<td>751 Ð 900</td>
<td>I/I/I</td>
<td>3</td>
</tr>
</tbody>
</table>

2. Let the class number be 5 and with the class interval as 150, we fill up the table.

3. Let 1. sq. unit = 30 taka along the x axis and 3 sq. unit = 1 unit along the y axis be taken.

4. We draw rectangles in each range having the height equal to the frequency of the class. In the diagram, the base of the first rectangle is bounded by 150 and 300 and its height is equal to \( 4 \times 3 = 12 \) units which indicates the frequency 4.

5. We draw 5 rectangles in the same way.

6. Lastly we mark the rectangles by oblique lines.
(b) Column diagram

Problem

The statistics of death by accident of Dhaka city in the first 10 days in a month are given below: Draw a column diagram.

<table>
<thead>
<tr>
<th>Date</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Death number</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

Procedure

1. We plot the dates at a distance of 5 sq. unit along the x axis in a graph paper.
2. Let 5 sq. unit = 1 unit along the y axis.
3. Then we put the death number along the y axis according to scale.
4. First we mark the point of death number of a date and then we join the x axis of the same date.
5. In this way we make ten columns of ten days.
N. B.: Since nobody died on the 9th day so the length of the column is zero. The length of the column of each day is proportional to the death number of the same day.

(c) Frequency polygon
The graph of a frequency polygon is similar to the system of column diagram in making the equal range of the class. But the mean of each class is used as variable.

<table>
<thead>
<tr>
<th>Problem No. 3.8</th>
<th>Date:</th>
</tr>
</thead>
</table>

Problem
The monthly wages including overtime of 30 workers in a factory are given below. Draw a frequency ploygon.


Theory:
By observation \( x_1 = 152, \ x_2 = 885 \).

Let the class interval \( h = 150 \)

\[
\therefore \text{The class number } k = \frac{885 - 152}{150} = 4.886
\]

In round figure \( k = 5 \) (next whole number)

Procedure
1. Let the range be filled up by taking the class interval as 150 and class number as 5.

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>Class mean (x)</th>
<th>Tally</th>
<th>Frequency (f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>151  ( \text{Ð} ) 300</td>
<td>225</td>
<td>( \overline{\text{IIII}} )</td>
<td>4</td>
</tr>
<tr>
<td>301 ( \text{Ð} ) 450</td>
<td>375</td>
<td>( \overline{\text{II}} )</td>
<td>7</td>
</tr>
<tr>
<td>451 ( \text{Ð} ) 600</td>
<td>525</td>
<td>( \overline{\text{III}} )</td>
<td>11</td>
</tr>
<tr>
<td>601 ( \text{Ð} ) 750</td>
<td>675</td>
<td>( \overline{\text{III}} )</td>
<td>5</td>
</tr>
<tr>
<td>751 ( \text{Ð} ) 900</td>
<td>825</td>
<td>( \overline{\text{III}} )</td>
<td>3</td>
</tr>
</tbody>
</table>
2. Let us consider 1 sq. = 15 units along x axis and 5 sq. = 1 unit along y axis.
3. x axis represents the amount of money and y axis represents the frequency.
4. Then we plot the point (75,0), (225,4), (375,7), (525,11), (675,5), (825,3) and (975,0) in graph paper.
5. Let the point be A, B, C, D, E, F, G respectively.

6. Joining the points we draw the frequency polygon ABCDEFG.

(N. B: Since there is no data between zero to 150, the graph may be started from (225, 4). But in the case, the first point (225,4) is to be joined with the class mean 75 of the former class 225 on X axis which denotes A. Similarly the last point (825,3) is to be joined with the class mean 975 of the latter class.)
(D) PIE GRAPH

Problem No. 3.9

Problem

The monthly expenditure of a family is given below. Draw a pie graph.
Food Tk. 2000.00  
Cloth Tk. 500.00  
House rent Tk. 500. 
Medical treatment Tk. 750.00 
Education Tk. 500.00  
Others Tk. 250.00

Theory :

\[ \theta = \frac{f}{N} \times 360^\circ \text{ [use protractor for measuring angles]} \]

Procedure

1. Let us compose the following table with the given data.

<table>
<thead>
<tr>
<th>Item of Expenditure</th>
<th>Exp. for each item (Tk) (f)</th>
<th>Total Exp. (In Tk. (N))</th>
<th>Angle at the centre in degrees ((\theta))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>2000</td>
<td>4500</td>
<td>160°</td>
</tr>
<tr>
<td>Cloth</td>
<td>500</td>
<td></td>
<td>40°</td>
</tr>
<tr>
<td>House rent</td>
<td>500</td>
<td></td>
<td>40°</td>
</tr>
<tr>
<td>Medical treatment</td>
<td>750</td>
<td></td>
<td>60°</td>
</tr>
<tr>
<td>Education</td>
<td>500</td>
<td></td>
<td>40°</td>
</tr>
<tr>
<td>Others</td>
<td>250</td>
<td></td>
<td>20°</td>
</tr>
</tbody>
</table>

2. We draw a circle taking any radius and let the centre be O and AO be any radius.
Diagram Ï 14

3. Now we draw \( \angle AOB = 160^\circ \) for food, \( \angle BOC = 40^\circ \) for cloth, \( \angle COD = 40^\circ \) for house rent, \( \angle DOE = 60^\circ \) for medical treatment, \( \angle EOF = 40^\circ \) for education and then the remaining \( \angle FOA \) at the centre of the circle must be equal to 20° which is for others.

Problem No. 3.10 Ï 3.11

Date :

**Problem**

(a) The passing percentages of students of a school in secondary examination from 1991 Ï 1995 are given below: Show them in bar diagram.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage of passing</td>
<td>90</td>
<td>85</td>
<td>92</td>
<td>95</td>
<td>97</td>
</tr>
</tbody>
</table>

(b) The marks in the examination of Geography obtained by 100 students of a school are given below. Draw a frequency polygon.

<table>
<thead>
<tr>
<th>Marks</th>
<th>40 - 50</th>
<th>50 - 60</th>
<th>60 - 70</th>
<th>70 - 80</th>
<th>80 - 90</th>
<th>90-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of students</td>
<td>15</td>
<td>18</td>
<td>20</td>
<td>35</td>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>
Problem
(a) Ten exporters of Bangladesh export respectively 825, 935, 1020, 1125, 975, 1060, 1280, 990, 892, 1180 tons of prawn in a certain year. Draw a bar diagram.

(b) The weights of 50 men are given below (in kilogram). Draw a bar diagram:

30, 35, 32, 30, 40, 31, 31, 45, 36, 44, 47, 37, 49, 47, 51, 52, 35, 40, 39, 48, 45, 52, 43, 37, 46, 48, 42, 51, 43, 53, 55, 47, 37, 38, 39, 50, 45, 39, 12, 43, 45, 47, 35, 38, 39, 46.

Problem
(a) The following are the number of books sold in 6 days of a week in a shop. Draw a columnar diagram.

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>90</td>
<td>85</td>
<td>70</td>
<td>98</td>
<td>80</td>
<td>95</td>
</tr>
</tbody>
</table>

(b) The week periods of a school is given subject wise. Draw a pie diagram.

<table>
<thead>
<tr>
<th>Subject</th>
<th>Bengali</th>
<th>English</th>
<th>Mathematics</th>
<th>Science</th>
<th>S. Sciene</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period</td>
<td>4</td>
<td>8</td>
<td>5</td>
<td>6</td>
<td>3</td>
<td>10</td>
</tr>
</tbody>
</table>

Problem
(a) The average prices of sugar per kilogram (month-wise) in a year are given below. Draw a columnar diagram.

<table>
<thead>
<tr>
<th>Month</th>
<th>January</th>
<th>February</th>
<th>March</th>
<th>April</th>
<th>May</th>
<th>June</th>
<th>July</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>23.50</td>
<td>23.00</td>
<td>23.75</td>
<td>24.35</td>
<td>24.00</td>
<td>25.25</td>
<td>25.75</td>
</tr>
<tr>
<td>August</td>
<td>26.00</td>
<td>26.25</td>
<td>27.75</td>
<td>23.75</td>
<td>25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(b) 24 best players are selected for training to make the national team from the different parts of Bangladesh. It is found that 7 are taken from Dhaka division, 4 from Chittagong, 5 from Rajshahi, 4 from Khulna, 1 from Barisal and 3 from Syllhet division. Draw a pie diagram.

<table>
<thead>
<tr>
<th>Problem No. 3.18 ð 3.19</th>
<th>Date :</th>
</tr>
</thead>
</table>

**Problem**

(a) The government of Bangladesh allotted the following in the budget in a year. Draw a pie graph.

Food = 20%, Health and treatment = 25%, Education = 30%, Communication = 15% and others = 10%.

(b) Of the total population of the world 20% live in America, 28% in Europe, 35% in Asia, 15% in Africa and 5% in Australia. Draw a pie graph (Continent wise).

3.3 Mean, Median and Mode

<table>
<thead>
<tr>
<th>Problem No. 3.20</th>
<th>Date :</th>
</tr>
</thead>
</table>

**Problem**

The daily wages of 50 labours working in the road constructions given below (in taka). Find out the mean, median and mode.


(a) **Process of finding mean**

Theory of first table
Let the lowest number of the data = $x_1$ and the highest number = $x_2$
The range = $x_2 - x_1$
Let the class interval = $h$
Then the class number $k = \frac{x_2 - x_1}{h}$
Procedure

1. From the given data \( x_1 = 90, \ x_2 = 180 \)
2. Let the class interval \( h = 15 \)
3. Class number \( k = \frac{x_2 - x_1}{h} = \frac{180 - 90}{15} = 90 \frac{15}{15} = 6 \)
4. Starting from 88 with 15 as class interval tally, class number, frequency, class -mid point, and added frequency are shown in the following chart :

<table>
<thead>
<tr>
<th>Class interval</th>
<th>Tally</th>
<th>Frequency (f)</th>
<th>Class mid point</th>
<th>Added frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>88 Ð 102</td>
<td>II</td>
<td>6</td>
<td>95</td>
<td>6</td>
</tr>
<tr>
<td>103 Ð 117</td>
<td>III</td>
<td>7</td>
<td>110</td>
<td>13</td>
</tr>
<tr>
<td>118 Ð 132</td>
<td>IIIII</td>
<td>9</td>
<td>125</td>
<td>22</td>
</tr>
<tr>
<td>133 Ð 147</td>
<td>IIIII</td>
<td>10</td>
<td>140</td>
<td>32</td>
</tr>
<tr>
<td>148 Ð 162</td>
<td>IIIII</td>
<td>8</td>
<td>155</td>
<td>40</td>
</tr>
<tr>
<td>163 Ð 177</td>
<td>III</td>
<td>6</td>
<td>170</td>
<td>46</td>
</tr>
<tr>
<td>178 Ð 192</td>
<td>I</td>
<td>4</td>
<td>185</td>
<td>50</td>
</tr>
<tr>
<td>( \sum f = n = 50 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. **Theory for finding the mean** : Let the mean be \( \bar{x} \)

The mean class interval of all class numbers = a. Class interval = h,

Class number = k

\[ \bar{x} = a + h \bar{u}, \text{ or } \bar{u} = \frac{x - a}{h} \]
6. Now we compose a table with the help of above theory.

<table>
<thead>
<tr>
<th>Class interval</th>
<th>Frequency (f)</th>
<th>Class midpoint (x)</th>
<th>a</th>
<th>$u = \frac{x-D}{h}$</th>
<th>$\bar{u}$</th>
<th>$\bar{\bar{u}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>88 Æ 102</td>
<td>6</td>
<td>95</td>
<td></td>
<td>D3</td>
<td>D18</td>
<td>6</td>
</tr>
<tr>
<td>103 Æ 117</td>
<td>7</td>
<td>110</td>
<td></td>
<td>D2</td>
<td>D14</td>
<td>13</td>
</tr>
<tr>
<td>118 Æ 132</td>
<td>9</td>
<td>125</td>
<td></td>
<td>D1</td>
<td>D9</td>
<td>22</td>
</tr>
<tr>
<td>133 Æ 147</td>
<td>10</td>
<td>140</td>
<td>140</td>
<td>0</td>
<td>0</td>
<td>32</td>
</tr>
<tr>
<td>148 Æ 162</td>
<td>8</td>
<td>155</td>
<td></td>
<td>1</td>
<td>8</td>
<td>40</td>
</tr>
<tr>
<td>163 Æ 177</td>
<td>6</td>
<td>170</td>
<td></td>
<td>2</td>
<td>12</td>
<td>46</td>
</tr>
<tr>
<td>178 Æ 192</td>
<td>4</td>
<td>185</td>
<td></td>
<td>3</td>
<td>12</td>
<td>50</td>
</tr>
</tbody>
</table>

7. Now we write the theory for finding means.

$$\text{Mean} (\bar{x}) = a + \frac{\sum f_i u_i}{n} \times h$$

8. According to theory,

$$\text{Mean} (\bar{x}) = 140 + \frac{(D9)}{50} \times 15 = 140 \ AE 2.7 = 137.3$$

(b) Method of finding median

1. First we do the similar work from the theory to the composition of first table. However, there will be no needed of class midpoint diagram to find out the median/mode.

2. Next theory and procedure:

The number which indicates median = $\frac{n}{2}$

The lower limit of median class = $L_m$

The class extent of median class = $h_m$

The frequency of median class = $f_m$

The added frequency of the class before the median class = $F_m$

$$\text{Median} = L_m + \frac{n}{2} \frac{DF_m}{f_m} \times h_m$$
3. We find the median according to the above theory. Here, the added frequency of class three is 22 and that of class four is 32.

∴ The median, which the value of \( \frac{n}{2} = 25 \) th datum, included in class four

\[ L_m = 133 \] (lowest boundary of class four)
\[ h_m = 15 \]
\[ f_m = 10 \] (frequency of median class)
\[ F_m = 22 \]

\( \therefore \) Median = 133 + \( \frac{50 \times 22}{10} \times 15 \) = 133 + \( \frac{3 \times 15}{10} \)

\[ = 133 + 4.5 = 137.5 \]

(C) Method of finding mode
1. First we do similar work from the theory to the composition of first chequer

2. Next theory and procedure
   Let the lower limit of mode class = \( L \)
   Class interval of mode class = \( h \)
   The difference between the frequency of mode class and its previous class = \( \Delta_1 \)
   The difference between the frequency of mode class and its next class = \( \Delta_2 \).
   Then the mode = \( L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h \)

3. By observation, we see that the highest number of frequency is in 133-147. Therefore the mode remains in that class i.e. in 133 \( \Delta \) 147.

4. Here we determine \( L = 133. h = 15, \Delta_1 = 10 \Delta 9 = 1,\Delta_2 = 10 \Delta 8 = 2 \)

5. Therefore, the mode = \( 133 + \frac{1}{1+2} \times 15 = 133 + 5 = 138 \)

(d) Method of determining mode from Bar diagram
1. First we make the No. 1 chequer from the given data.

2. We find in No. 1 chequer that the number of frequency is the highest in 133 \( \Delta \) 147.
3. In a piece of graph paper we take 5 sq. = 15 units i.e. 1 sq. = 3 units along the x axis.

4. Then we take 5 sq. = 1 unit along the y axis.

5. We consider wages along the x axis and labours along the y axis and

6. The height of 133 Ð 147 is the most highest and hence the mode remains in this class.

7. Now we join the right vertex of this rectangle with the right vertex of the left side rectangle and the left vertex of this rectangle with the left vertex of the right side rectangle.

8. These lines intersect at P.

9. From the point P we draw PM ⊥ to x axis.

∴ The position of M indicates the mode.
Calculation: Here, the point M is found at a distance of 1.5 sq. from the position of 133.

∴ Mode = 133 + 1.5 × 3 = 133 + 4.5 = 137.5

Problem

The marks of Higher Secondary examination obtained by 120 students who are seeking admission into the honours course of the Dhaka University are given below. Find the mean, median and mode of them and also find the mode by drawing a bar diagram.

<table>
<thead>
<tr>
<th>Marks obtained</th>
<th>Number of candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>501 ß 550</td>
<td>13</td>
</tr>
<tr>
<td>551 ß 600</td>
<td>30</td>
</tr>
<tr>
<td>601 ß 650</td>
<td>40</td>
</tr>
<tr>
<td>651 ß 700</td>
<td>18</td>
</tr>
<tr>
<td>701 ß 750</td>
<td>12</td>
</tr>
<tr>
<td>751 ß 800</td>
<td>7</td>
</tr>
</tbody>
</table>

Problem

The daily average income of 30 rickshaw pullers is given below. Find the mean, median and mode of the data and also find the mode by means of bar diagram.

Problem No. 3.22

To present the given data by pie graph.
The weekly subject-wise lesson programmes of a school were as follows.

<table>
<thead>
<tr>
<th>Subject</th>
<th>Bengali</th>
<th>English</th>
<th>Mathematics</th>
<th>Science</th>
<th>S. Science</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson hour</td>
<td>4</td>
<td>8</td>
<td>5</td>
<td>6</td>
<td>3</td>
<td>10</td>
</tr>
</tbody>
</table>

The given data are to be presented by pie graph.

**Theory:**

\[ f = \text{Time for the lesson in each subject (in hour)} \]
\[ N = \text{Total time for lesson (in hour)} \]
\[ = (4 + 8 + 5 + 6 + 3 + 10) \text{ hour} = 36 \text{ hour} \]
\[ \therefore \text{ The product angle, } \theta = \frac{f}{N} \times 360^\circ \]

**Materials**

1. A sharpen pencil
2. Protractor
3. Scale
4. Pencil compasser
5. Rubber
6. Graph paper etc.

**Procedure**

1. We compose the following chart with the given data:

<table>
<thead>
<tr>
<th>Subject</th>
<th>Lesson (in hour)</th>
<th>Total (in hour)</th>
<th>Angle (in degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bengali</td>
<td>4</td>
<td></td>
<td>40°</td>
</tr>
<tr>
<td>English</td>
<td>8</td>
<td></td>
<td>80°</td>
</tr>
<tr>
<td>Mathematics</td>
<td>5</td>
<td>36</td>
<td>50°</td>
</tr>
<tr>
<td>Science</td>
<td>6</td>
<td></td>
<td>60°</td>
</tr>
<tr>
<td>S. Science</td>
<td>3</td>
<td></td>
<td>30°</td>
</tr>
<tr>
<td>Others</td>
<td>10</td>
<td></td>
<td>100°</td>
</tr>
</tbody>
</table>
2. We draw a circle of any radius with centre O and any radius AO be drawn.

3. Now we draw $\angle AOB = 40^\circ$ for Bengali, $\angle BOC = 80^\circ$ for English, $\angle COD = 50^\circ$ for Mathematics, $\angle DOE = 60^\circ$ for Science, $\angle EOF = 30^\circ$ for Social Science and $\angle FOA = 100^\circ$ for others.

**Precautions**

1. We are to be careful to draw the angles by protractor.
2. We are to indicate the different subjects carefully.
CHAPTER FOUR
GEOMETRICAL DRAWING

4.1 PROPORTIONAL CONSTRUCTION

The measurement of areas of different shapes of geometric figures or the volume of solid figures can be solved by means of mathematical formula in practical life. In some cases the area or the volume cannot be measured without proper construction of diagrams. Hence a proportional diagram is necessary for a larger figure with a certain scale. For example, a rectangular land having a length of 207 m and breadth of 153 m cannot be drawn in a simple piece of paper for which 10 m = 1 cm may be used as a scale in this construction. This 1 cm = 10 m is called one kind of scale. Moreover 20m = 1 cm may also be needed. It is to be remembered that the length smaller than \( \frac{1}{10} \) th of a centimeter will not be possible to measure with the scale we generally use. If we consider the scale 1 cm = 20 m then 207 m will be equal to 10.35 cm. But it will not be easy to measure 10.35 cm correctly. So in this case if we consider the scale 1 cm = 10 m then proportional length and breadth of the above stated land will be 20.7 cm and 15.3 cm respectively. Whatever scale is used must be mentioned right below the diagram. Selection of scale, drawing and method of noting in the practical khata are shown in the next example. The following objects must be kept with one for any geometrical construction:

1. A sharpened wood pencil.
2. A pencil cutter and a piece of eraser.
3. A ruler of minimum 6 inches long having scales of inch and centimeter.
4. A protractor which may be used with proper instruction.
5. A pencil compasses and a divider.
6. A fountain pen or a ball-point pen.

Some particular instructions for method of construction

To draw any geometrical figure some particular data are given, e.g. two sides and an angle of a triangle or the parallel sides and the adjacent angles of a
trapezium or an angle, straight line according to given data which are to be
drawn in the left or right side of the paper and then the required diagram is to be
drawn accordingly.

Some examples are shown in which the method of construction, the diagrams
and the systems of writing a practical note book in the class room. Some
problems are given with indications only which are to be drawn according to the
examples given in the practical note book. To obtain full marks in the practical
note book it is necessary to be regular in class work having neat and clean
diagrams with proper description.

<table>
<thead>
<tr>
<th>Problem No. 4.1</th>
<th>Date :</th>
</tr>
</thead>
</table>

**Problem**

A proportional diagram is to be drawn of a triangular field having the sides of
lengths 65 m, 57m and 50 m.

**Theory :**

If the lengths of three sides of a triangle be given, then a unique triangle can be
drawn.

**Necessary materials**

A sharpen wood pencil, scale, pencil
compasses, rubber.

**Construction :**

1. Let the scale be 1 cm = 10 m.

   The sides of the proportional
   triangle will be
   6.5 cm, 5.7 cm, and 5 cm.

2. By scale we draw three line
   segments a = 6.5 cm., b = 5.7
   cm.,c = 5 cm.
3. We cut BC = a from the line segment BD and with the centre B, draw an arc of radius c.
4. Again with the centre C, draw an arc of radius b.
5. The above two arcs intersect each other at the point A.
6. Join AB and AC. Then the triangle ABC will represent the proportional diagram of the given piece of land.

| Problem No. 4.2 | Date : |

**Problem :**

1. The length of the side of a square is 68 m. There is an isosceles triangle attached on one of its extended side of the square having a height of 44 m. To draw the proportional diagram of the whole figure.

2. The length and breadth of a garden is 63 m and 35 m respectively. To draw a proportional diagram of the road whose width is 3.5 m within the inner four sides of the garden. To identify the road separately and also to find out the area of the road.

3. To draw a proportional diagram of an equilateral triangle of which the length of the side is 108 m.

4. The length of the side of an equilateral triangular field is 48 m. A square is drawn on any one side of the triangle outside of it. To draw the proportional diagram of the whole field.

5. A land in the shape of a quadrilateral has one of its diagonals of length 130 m. On one of its sides the lengths of the sides are 120 m and 50 m and on the other side the length of the sides are 100 m and 70 m. To draw a proportional diagram of the land.
4.2 Geometrical drawing relating to triangles, quadrilaterals and circles.

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**Problem**

To draw a triangle having given the base, difference of two base angles, the sum of the other two sides.

**Construction:**

1. Let \( a \) be the base, \( \angle X \) be the difference of two base angles and 's' be the sum of other two sides;
2. Let \( BC = a \) is cut from any straight line \( BD \).
3. At \( C \) on \( BC \), \( \angle BCE = \frac{1}{2} \angle x \) is drawn.
4. \( CF \) is drawn perpendicular on \( CE \) at the point \( C \).
5. An arc is drawn with a radius equal to 's' at the point \( B \) such that it intersects \( CF \) at the point \( G \).
6. Now at \( C \), \( \angle GCA = \angle CGB \) is drawn such that \( CA \) cuts \( BG \) at \( A \). Then \( \triangle ABC \) is the required triangle.

**Proof:** \( \angle ECG = 90^\circ \)

\[
\therefore \quad \angle AEC + \angle AGC = 90^\circ \\
\text{Again} \quad \angle AGC = \angle ACG \text{ (As per drawing)} \\
\therefore \quad \angle ACE = \angle AEC \\
\text{Now,} \quad \angle ACB \triangle \angle ABC = (\angle ACE + \angle BCE) \triangle (\angle AEC \triangle \angle BCE) \\
= \angle ACE + \angle BCE \triangle \angle AEC + \angle BCE \\
= \angle BCE + \angle BCE \\
= 2\angle BCE \text{ (as} \angle ACE = \angle AEC) \\
= 2 \cdot \frac{1}{2} \angle x = \angle x \\
\text{Besides,} \quad \angle AGC = \angle ACG \\
\therefore \quad AC = AG \\
\therefore \quad BG = BA + AG = BA + AC = s \\
\therefore \quad \triangle ABC \text{ is the required triangle.}
Problem:

To draw the triangles with the following data: (to draw the line segment by scale and the given angles by protractor.)

1. Base 14 cm. the difference of the base angles = 20° and the sum of the two other sides 16.8 cm.
2. Base 16 cm. the difference of the base angles = 30° and the sum of the two other sides = 21.5 cm.
3. Base = 8 cm, the difference of the base angles = 15° and the sum of the two other sides = 22 cm.

Problem

To draw a triangle having given the base, the difference of the base angles and the difference of the other two sides.

[Hints: Let 'a' be the length of the base, ∠x be the difference of the base angles and 'd' be the difference of other two sides.]

1. BC = 'a' is cut from any straight line BD and draw ∠BCE = \( \frac{1}{2} \) ∠x at C.
2. With centre B and radius equal to d, draw an arc of a circle which cuts BC at M and N.
3. Join BN and extend it up to F.
4. Draw ∠NCA = ∠FNC at C and then ABC will be the required triangle.
Problem
To draw the triangles with the following data: (to draw the line segment by scale and the given angles by protractor.)
1. Base = 19.6 cm, the difference of the two base angles = 30° and the difference of the two other sides = 11.7 cm.
2. Base = 14 cm, the difference of the two base angles = 20° and the difference of the two other sides = 11.0 cm.
3. Base = 15 cm, the difference of the two base angles = 15° and the difference of the two other sides = 9.0 cm.

Problem
To construct a triangle having given the base, the vertical angle and the ratio of other two sides.

[Hints: Let 'a' be the length of the base, $\angle x$ be the vertical angle and m : n be the ratio of other two sides.]
1. Let m and n be the two straight lines and BP = m is cut from any straight line BD.
2. $\angle BPE = \angle x$ is drawn at the end P on BP and PR = n is cut from PE.
3. Join BR and BR is extended up to F. Take BC = a from BF.
4. At C, CA $\parallel$ BP is drawn so that CA cuts BD at A. Then ABC will be the required triangle.

Problem
To draw the triangles with the following data: (to draw the given line segment by scale and the given angles by protractor.)
1. Base = 12.5 cm, the vertical angle = 30° and the ratio of the two other sides = 5:7.
2. Base = 9.4 cm, the vertical angle = 80° and the ratio of the two other sides = 7 : 10.

3. Base = 14.8 cm, the vertical angle = 45° and the ratio of the two other sides = 12:13.

**Problem**

To construct an isosceles triangle so that each of its base angles is twice the vertical angle.

**Construction**:

1. Any straight line AB is taken and it is bisected at O.

2. BP is drawn perpendicular on AB at the point B.

3. BQ = 1/2 AB is cut from BP and AQ is joined.

4. QR = 1/2 AB is cut from QA and AD = AR is cut from AB.

5. With centres B and D and radius equal to AD, two arcs are drawn on the opposite side of AQ of the side AB which cut each other at C; AC is joined. Then ΔABC will be the required triangle.

**Proof**:

The circumcircle of ΔADC is drawn. Now AD = CD = BC (by construction).

∴ \( \angle ACD = \angle CAD \)
Again $AD^2 = AB.BD = BC^2$ (by construction).

∴ $BC$ is the tangent of circle $ACD$

∴ $\angle BCD = \angle CAD$ [angle at the alternate segment]

Again $\angle BDC = 2 \angle CAD = 2 \angle BAC$

∴ $BD = CD$

$\angle BDC = \angle DBC$

or, $2 \angle BCD = \angle DBC = \angle ABC$

Again $\angle ACB = \angle ACD + \angle DCB$

$= \angle CAD + \angle CAD = 2\angle CAD$

$= 2\angle BAC = \angle ABC$

∴ $AB = AC$

∴ $\Delta ABC$ is an isosceles triangle.

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**Problem**

To draw a triangle having given the base, one of the base angles and the sum of the other side of the angle and the height of the triangle.

[Hints : Let 'a' be the given base, $\angle x$ be the base angle and 's' be the sum of the height and the other side of the given angle.

1. Take a point 'P' on any straight line $XY$ and draw $PQ$ perpendicular to $XY$ at the point 'P'.

2. Cut $PD = s$ from $PQ$ and draw $\angle PDB = \frac{1}{2}$ of the complementary $\angle x$ at $D$ so that $DB$ cuts $XY$ at $B$.

3. Again draw $\angle DBA = \angle PDB$ at $B$ so that $BA$ cuts $PD$ at $A$.

4. Cut $BC = 'a'$ from $BY$ and join $AC$. Then $ABC$ will be the required triangle.]
**Problem**

To draw the following triangles with the given data:

1. The base = 14 cm; one of the base angles = 60° and the sum of the height and the other side of the given angle is 24.8 cm.
2. The base 16 cm; one of the base angles = 45° and the sum of the height and the other side of the given angle is 21.0 cm.
3. The base 15 cm; one of the base angles = 30° and the sum of the height and the other side of the given angle is 19.8 cm.

**Problem**

To draw a square equal in area to a given quadrilateral.

[Hints:]

1. Take any quadrilateral ABCD and draw the triangle ADX equal in area to ABCD.
2. Draw a perpendicular DP on AX from D at the point 'P'
3. Find the mid point O on DP and draw YZ \parallel AX through O.
4. Draw AL and BM perpendicular to YZ from A and X which cut YZ at L and M respectively. Then the area of the rectangle ALMX will be equal to the area of ABCD.
5. To draw a square whose area will be equal to the area of the quadrilateral ALMX.
6. Produce LM up to E such that ME = MX.
7. Draw a semicircle taking LE as diameter.
8. Produce XM such that it intersects the semi-circle at N. Then MN is the side of the required square.
9. Draw a square MNUV taking MN as one of the sides of the square.
Then MNUV is the required square whose area is equal to the area of the quadrilateral ABCD.

Problem No. 4.13  
Date : 

**Problem**  
To construct the square equal in area to a given quadrilateral in each case having the base 14 cm when one of the base angles is (a) 60°, (b) 30°, (c) 45°.

Problem No. 4.14  
Date : 

**Problem**  
To construct an equilateral triangle equal in area to a given square.

1. Take any square ABCD and produce AB to E so that AB = BE.
2. Draw two arcs one at A and another at E taking AE as radius so that the two arcs intersect at the point F.
3. Join AF and EF such that AF intersects the line DC at the point H.
4. Find the mean proportion of AH and AF. Let the mean proportion is equal to 'd' so that \( d^2 = AH \cdot AF \).
5. Cut AP = d from AF and draw PQ \parallel EF so that PQ cuts AE at Q. Then APQ will be the required triangle.

Problem No. 4.15  
Date : 

**Problem**  
Draw the equilateral triangle each of whose area will be equal to the area of square having the sides (a) 12.7 cm, (b) 13.5 cm, (c) 15 cm and (d) 14.6 cm.

[The sides of the equilateral triangle will be equal to the length of the sides of the square in four different cases.]
Problem
To construct a parallelogram with a given angle having the area equal to the area of a given triangle.

Materials
Pencil, Compasses, Scale etc.

Construction
1. Let $\Delta ABC$ be any triangle.
2. Bisect BC at E.
3. Draw $\angle CEF = \angle x$ at the point E.
4. Draw $\text{AFG \parallel BC}$ through A so that it cuts EF at F.
5. Draw $\text{CG \parallel EF}$ through C so that it cuts the line AFG at G.

Then $\text{ECGF}$ will be the required parallelogram.

Proof:
Join A, E.

Since the area of $\Delta ABE = \text{the area of } \Delta ACE$

$\therefore \text{\Delta-region } ABC = 2\Delta\text{-region AEC}.$

$\therefore \text{ECGF} = 2\Delta\text{-region AEC}.$

$\therefore \text{ECGF} = \Delta ABC$

Moreover, $\angle CEF = \angle D$ (As per drawing)

$\therefore \text{ECGF is the required parallelogram.}$
Problem

To construct by adopting the following data:

1. Construct a parallelogram with an angle equal to 60° having the area equal to an isosceles triangle of which the base = 15.6 cm and height = 8.3 cm.

2. Construct a parallelogram with an angle equal to 30° having the area equal to a given triangle of which the sides are 5 cm, 6 cm and 7 cm.

3. Construct a parallelogram with an angle equal to 75° having the area equal to a triangle of which the base = 14 cm; the difference of the base angles = 20° and the sum of other two sides = 16 cm.

Problem

To draw a trapezium having given the lengths of two parallel sides a and b (a > b) and the two angles ∠x and ∠y adjacent to a.

[Hints:

1. Take any line BC (= a)
2. Draw ∠CBM = ∠x and ∠BCN = ∠y at B and C respectively on the line BC.
3. Let BM and CN intersect at P. Draw PQ || BC at P.
4. Take PR (= b) from the line PQ.
5. Draw RD || BM at the point R so that RD cuts CN at D.
6. Draw DA || CB through D so that DA cuts BM at A.

Then ABCD will be the required trapezium.]

Problem

To construct trapeziums with the followings:
1. Lengths of the parallel sides are 5.6 cm and 8.5 cm. and two angles are 30° and 60° adjacent to the greater side.

2. Lengths of parallel sides are 7.0 cm and 5.0 cm and two angles are 60° and 30° adjacent to the greater side.

Problem
To construct a parallelogram having an angle equal to $\angle x$ and a side equal to 'd' and whose area is equal to the area of a triangle.

[Hints :

1. Let $ABC$ be a triangle and cut $BE = 2d$ from $BC$ (produce $BC$ if necessary)

2. Draw such kind of $\triangle PBE$ on $BE$ so that $\angle CBP = \angle x$ the and $\triangle PBE = \triangle ABC.$

3. Draw $\square EDGH = \Delta -$region $PBE.$

Then $\square EDGH$ will be the required parallelogram.]

Problem
To construct parallelogram with an angle equal to 30° and a side equal to 4.2 cm and the areas equal to the areas of the following triangles.

(a) A triangle having the three sides 8.5 cm, 6 cm and 7 cm.

(b) A triangle having the base 4.2 cm and the height 2.3 cm.

(c) A triangle having the base 14 cm, the difference of the base angles 20° and the sum of the other two sides 16 cm. (Consider three different problems).
Problem
To construct a circle having given the radius $r$ which will touch a given circle and a given straight line.

Necessary Materials
Pencil, Compasses, Scale etc.

Construction

1. Let $AB$ be a straight line. At a distance $r$ from $AB$, draw $EF \parallel AB$. So the centre of the required circle will lie on $EF$.

2. Let 'C' be the centre of a circle whose radius is $R$. Let us draw a circle with radius $R$ and centre at $C$. This is the given circle.

3. Taking radius equal to $R + r$ with centre $C$ draw a circle so that it intersects $EF$ at and $O$ and $O\prime$.

4. Draw two circles with the radius $r$, one with centre $O$ and another with the centre $O\prime$.

Then both the circles are the required circles.

Proof:
Join C, B.
Since $AC \perp AB$ and $KO \parallel AB$ and $KA = OB = r$,
Therefore, $KABO$ is a rectangle.
$\therefore \ OB \perp AB$.
$\therefore$ The required circle will touch $AB$ at $B$. 
Again CO = CD + r and the points C, D and O are on the same straight line.

∴ The required circle will touch the circle whose centre is at C.

Similarly, it can be shown that the circle with centre at O also touches the given circle and the given straight line.

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**Problem**

1. To draw a circle having given the radius 3 cm which will touch a given straight line and a given circle with a radius of 5 cm.

2. To draw a circle having given the radius 2.5 cm which will touch a given straight line and a given circle with a radius of 3.7 cm.

3. To draw a circle having given the radius 1.8 cm which will touch a given straight line and given circle with a radius of 2.3 cm.

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**Problem**

To construct a circle which will touch a given circle at a given point on another given circle.

**[Hints]**

1. Draw a circle having the centre O and take any point A on the circumference.

2. Draw another circle with the centre at C.

3. Join O, A and produce it up to P.

4. Draw a tangent AX at the point A on the circle whose centre is at 'O'.

5. Draw CD \( \perp \) AX at the point C which meets D.

6. Produce DC which cuts the circle with the centre at O at Q. Join QA so that QA cuts the same circle at R.
7. Join C, R and produce CR which intersects produced PA at S. Then S will be the centre of the required circle.

8. With centre S and radius SA or SR draw the circle that will be required circle.

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**Problem**

To draw a circle which will touch a given circle at a given point and a given straight line.

**Hints**

1. Let AB be any straight line and draw a circle having the centre O and P be any point on the circumference of the circle.
2. Draw OX \perp AB and produce AX so that it cuts the given circle at R and S.
3. Join R, P and S, P and produce them so that they cut AB at E and F respectively.
4. Draw EH \perp AB and FK \perp AB. Join OP and produce OP so that it cuts EH and FK at H and K respectively. Then H or K will be the centre of the required circle and the radius will be HP or KP.
5. With centre H and K and radius HP and KP respectively, if two circles are drawn, they will represent the required circles.

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**Problem**

1. AB is a given straight line and a circle having the radius 3 cm is at a distance of 4 cm from AB and P is any point on it. To draw two circles which will touch the given circle at P as well as the given straight line AB.
2. With all conditions of Q. 1, to draw the circles taking the radius of the given circles, as (a) 2.5 cm, (b) 2.0 cm, (c) 1.5 cm.
3. With all conditions of Q.1, to draw circles at distances (a) 3.0cm, (b) 3.7 cm from AB.
Problem No. 4.27
Date :

Problem
To draw a circle which will pass through two given points and will touch a given circle.

[Hints :
1. Draw any circle CDE and take two points A and B outside of it.
2. Draw a circle through A and B so that it cuts the circle CDE at C and D.
3. Join CD and AB and produce them so that they cut one another at X.
4. Draw tangent XE from the point X to the circle DCE.

Then the circle passing through A, B, E will be the required circle.]

Problem No. 4.28
Date :

Problem
To construct a circle passing through two given points outside of it and touching the circles having the radius (a) 4 cm, (b) 2.5 cm, (c) 3.5 cm.

Problem No. 4.29
Date :

Problem
To construct a circle which will touch parallel straight lines and will pass though a point inside the parallel lines.

Construction
1. MN and XY are two parallel straight lines and A be the given point in between the lines.
2. Draw YH ⊥ XY at Y on XY which cuts MN at H.
3. QR is drawn perpendicular bisector of YH which cuts YH at Q.
5.1 For convenience of drawing some models of solid figures are given below.

Diagram Nº 22
5.2 Some formulae of finding areas of plane figures are given below:

(a) Area of a triangle = \( \sqrt{s(s-a)(s-b)(s-c)} \) sq. unit when the length of sides are \( a, b, c \) units and \( s = \frac{1}{2} (a + b + c) \). Again the area \( a \) of a triangle = \( \frac{1}{2} \times a \times h \) sq. unit when the length of base is 'a' unit and height is 'h' unit.

(b) Area of a rectangle = (Length \times breadth) sq. unit.

(c) Area of a square = (length)\(^2\) sq. unit.

(d) Area of a parallelogram = (Base \times altitude) sq. unit.

(e) Area of a Rhombus = \( \frac{1}{2} \times \text{(Product of diagonals)} \) sq. unit.

(f) Area of a trapezium = \( \frac{1}{2} \times (\text{Sum of the parallel sides} \times \text{height}) \) sq. unit.

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**Problem**

Draw the diagram of the following solid figures and make their models.
(a) Cube, (b) rectangular solid figure, (c) Cylinder, (d) Sphere, (e) Cone (f) Pyramid.

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**Problem**

The length of diagonal BD of a quadrilateral ABCD is 135 m, and the length of the perpendiculars from the two vertices, A, C upon the diagonal BD are 63 m. and 37 m. respectively. Length of the feet of the perpendiculars from B are 67 m. and 32 m. respectively. Draw a proportional diagram of the quadrilateral and also find the area of the quadrilateral field.
**Theory:**
Area of quadrilateral
\[ = \frac{1}{2} \text{ (Length of a diagonal} \times \text{ Sum of the perpendicular distances from other two vertices to the diagonal)} \]

**Necessary Materials:** Pencil, Scale.

**Construction:**
1. Let the scale be 1cm. = 20 m. Therefore the proportional lengths of diagonal and the perpendiculars shall be 6.75cm, 3.15 cm and 1.85 cm respectively. Lengths of the feet of the perpendiculars from B are 3.35 cm and 1.6 cm. respectively by scale.

2. Take a straight lines d = 6.75cm., i = 3.15 cm., p = 3.35 cm. and q = 1.6 cm,

3. Take any two points P and Q on BD and draw perpendiculars PM and QN on BD. From PM, cut PA = 3.15 cm and from QN, cut QC = 1.85 cm. Then ABCD will be the required proportional quadrilateral.

**Result calculation**
Area of \( \square \) ABCD
\[ = \text{area of } \Delta BAD + \text{area of } \Delta BCD \]
\[ = \frac{1}{2} \text{ BD} \times PA + \frac{1}{2} \text{ BD} \times QC \]
\[ = \frac{1}{2} \text{ BD (PA + QC)} \]
\[ = \frac{1}{2} \times 135 \ (63 + 37) \text{ sq. units} \]
\[ = \frac{1}{2} \times 135 \times 100 \text{ sq. m.} \]
\[ = 6750 \text{ sq. m.} \]
Problem

1. The length and breadth of a rectangular garden are 67 m, and 48 m. There is a path of 2 m. wide around the garden inside of it. Draw a proportional figure of the garden and find the area of the path.

2. ABCDE is a pentagon. The length of AC and AD are 125 m, and 105 m, respectively. The length of the perpendicular drawn from E upon the side AD is 45 m. and the length of the perpendiculars drawn from B and D upon AC are 35 m. and 30 m. respectively. Draw a proportional diagram of the pentagon and find its area.

3. ABCD is a quadrilateral having the base AB = 95 m. The length of the perpendiculars drawn from C and D upon AB are 35 m. and 45 m. respectively. The feet to the perpendiculars on AD are X and Y, the points X and Y divide the line AB into three parts such that AX = 32 m, XY = 35 m. and YB = 28 m. Draw a proportional diagram and find the area of the quadrilateral.

5.3 Measure of solid figures

(a) If the length, the breath and the height of a rectangular solid are a, b, c units respectively. Then,

(i) The volume of rectangle = abc cu. unit,

(ii) The area of whole surface = 2(ab + bc + ca) sq. unit.

(iii) The length of diagonal = \( \sqrt{a^2 + b^2 + c^2} \) unit.

Problem

The length, breadth and height of a rectangular wooden block are 0.52 m, 0.21 m, and 0.26 m. respectively. Draw a proportional diagram and find its volume, the length of its diagonal and the area of the whole surface.
Theory:

Let the length, breadth and height of the wooden block be $a$, $b$, $c$ respectively. Then, volume = $abc$ cu. unit, length of diagonal = $\sqrt{a^2 + b^2 + c^2}$ unit. Area of whole surface = $2(ab + bc + ca)$ sq. unit.

Construction:
1. Take a piece of graph paper and let 1 small sq. unit = 2 cm. = 0.02m. as scale.
2. Then the length, breadth and height of the wooden block will be 26 unit, 10.5 unit and 13 unit respectively.
3. ABCD parallelogram is drawn on the graph paper having $AB = 26$ sq. and $AD = 10.5$ sq.
4. Perpendiculars $AA_1$, $BB_1$, $CC_1$ and $DD_1$ are drawn at $A$, $B$, $C$, $D$ respectively having the length of each perpendicular 13 sq.
5. $A_1B_1$, $B_1C_1$, $C_1D_1$, $D_1A_1$ are joined. Hence the proportional diagram is drawn having the parallel surfaces ABCD and $A_1B_1C_1D_1$.
6. BD₁ and CA₁ are joined; then BD₁ and CA₁ represent the diagonals.
Result calculations:
Volume = \((0.52 \times 0.21 \times 0.26)\) cu. m. = 0.0284 cubic m.

Area of whole surface:
\[= 2(0.52 \times 0.21 + 0.21 \times 0.26 + 0.26 \times 0.52)\] sq m = 0.998 sq. m,

The length of the diagonal
\[= \sqrt{(0.52)^2 + (0.21)^2m + (0.26)^2} = 0.6181\] m.

Problem
1. The length, the breadth and the height of the outer side of a covered wooden box are 3.5 m., 2.5 m. and 2 m. respectively. The thick of the wood is 2.5 cm. Draw a proportional diagram of the box. Find the quantity of wood required for manufacturing the box. Find also the expenses of polishing the outside of the box at taka 155.87 per sq. m.
2. The area of the whole surface of a cube is 588 sq. m. Draw a proportional diagram of the cube and verify your construction by measuring the diagonal.
3. Draw a rectangular solid of which the length of it's diagonal is 14.9666 m. and the ratio of its length, breadth and height is 3: 2: 1. If an iron sheet of 1.5 cm. thickness is plated around the solid, what quantity of iron is required.

CIRCULAR CYLINDER
If the radius of the base be 'r' and height be 'h' of a circular cylinder then the area of it's curved surface = \(2\pi rh\) sq. unit. The area of the base \(\pi r^2\) sq. unit.

The Volume = \(\pi r^2 h\) cu. unit.
Area of the whole surface = \((2\pi r^2 + 2\pi rh)\) sq. unit
\[= 2\pi r (r + h)\] sq. unit.

Problem
The radius of the base of a circular cylinder is 55 cm. and the height is 135 cm.
Draw its proportional diagram and determine its volume and the whole area of the surfaces.

**Diagram D 25**

**Theory**
Let the radius of the base be 'r' and the height be 'h' of the cylinder. Then the area of the whole surface

\[ = (2\pi r^2 + 2 \pi rh) \text{ sq. unit.} \]

\[ = 2\pi r (r + h) \text{ sq. unit.} \]

and the volume \( = \pi r^2 h \text{ cu. unit.} \)

**Construction**
1. Let a piece of graph paper be taken and let the scale be 1 sq. = 10 cm.
2. Then considering the radius 55 cm = 5.5 sq. and the height 135 cm. = 13.5 sq. We take 5.5 sq. in both the sides of AB from point A. Let the point be P and Q.
3. Then at the height of 13.5 sq. from P and Q we fix up two points S and R.
4. PQ and SR are joined and we draw two circles as ellipse at the centre A and B with diameters PQ and SR respectively.
5. Join AB. Then AP or AQ will be the radius of the cylinder of which the height will be AB. Hence, PQRS will be the proportional diagram of the cylinder.
Theory:

Oblique height \( l = \sqrt{h^2 + r^2} \)

Area of the curved surface = \( \pi rl \)

Volume = \( \frac{1}{3} \pi r^2 h \)

Diagram – 26

Construction:

1. Take a piece of graph paper. Let the radius of the base and the height be 3.3 m. and 8.4 m. respectively.

2. Let the scale be 0.3 m. = 1 square of the graph paper. Hence the radius of the base will be 11 sq. and the height will be 28 sq. of the graph paper.

3. Let the line PQ = 22 sq. and bisect it at O.

4. Draw \( \text{AO} \perp \text{PQ} \) at O so that \( \text{AO} = 28 \) sq. long. A,P and A,Q are joined.

5. Draw a circle (as ellipse) taking PQ as the diameter. Then APQ will be proportional diagram of the cone. OA can be said as the height of the cone or axis of the cone, AP is the oblique height of the cone.
Result calculation

\[ r = 3.3 \text{ m., } h = 8.4 \text{ m} \]

\[ \therefore l = \sqrt{(3.3)^2 + (8.4)^2} \text{ m} = 9.0250 \text{ m}. \]

Area of the curved surface = \( \pi \times 3.3 \times 9.0250 \text{ sq. m.} = 93.5645 \text{ sq. m.} \)
Volume = \( \frac{1}{3} \pi \times (3.3)^2 \times 8.4 \text{ cu. m} = 95.7934 \text{ cu. m.} \)

Verification

In actual measurement \( AP = 30 \text{ sq. m} = 30 \times 3 \text{ m} = 9.0 \text{ m.} \)

But in the calculation \( AP = l = 9.0250 \text{ m.} \)

The measurements of \( l \) by construction and by formulae are same (approximately).

The construction is correct.

<table>
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<th>Problem No. 5.9</th>
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Problem

Draw a proportional diagram of the right circular cone when its radius of the base and the height are 18 m. and 22 m. respectively. Construct a proportional diagram of a circular cylinder whose area of the curved surface is equal to the area of the curved surface of the cone.

If the cone is solid made of lead then after melting the cone prepare a solid cube. Draw a proportional diagram of the same (Consider 1% of lead is lost during melting.)

Sphere : If the radius of the sphere be \( r \), then its

(i) Volume = \( \frac{4}{3} \pi r^3 \) cu. unit.

(ii) Area of the curved surface = \( 4\pi r^2 \) sq. unit.

(iii) The radius of the circle obtained from the cross-section of the sphere at a distance \( h \) (\( h < r \)) from the centre of the sphere = \( \sqrt{r^2 - h^2} \) unit.
Problem
A circle is made at the cross-section of a sphere at a distance of 10.5 m from the
centre of a sphere whose diameter is 34 m. Justify the radius of the figure by
finding the radius of that circle. If the sphere is made of lead and if three small
spheres are made from this (considering 1% of lead is diminished during melting)
then draw a proportional diagram of a new small sphere.

Theory :
It the radius be r, then the volume = \( \frac{4}{3} \pi r^3 \) and the radius of the circle made
from the horizontal cross-section of the sphere = \( \sqrt{r^2 - h^2} \), where the distance of
the plane from the centre is 'h'.

Construction
(i) Take a piece of graph paper and let the scale be 1 sq. = 1m.
(ii) The diameter of the sphere = 34 m. Hence a straight line is taken having
the length 34 sq.
(iii) Draw a circle taking AB as diameter whose centre is O.
(iv) Take a point N at a distance of 10.5 sq. from the point O. Join ON.
(v) A plane through N and perpendicular to ON intersects the sphere and a
circle is produced whose diameter is PQ. The centre of this circle is N. OP is joined. The figure represents the proportional diagram of the
sphere.

Result Calculation
The radius of the circle made from the cross-section.

\[ = \sqrt{(17)^2 - (10.5)^2} = 13.3697 \text{ m. From the diagram } PN = 13.3697. \]

Hence the construction is perfectly correct.
**Last part**: The volume of the lead of the sphere $= \frac{4}{3} \pi (17)^3$ cu. m.

$= 20579.5263$ cu. m.

Since 1% of the lead is diminished so the volume of the remaining lead

$= (20579.5263 - 205.795263)$ cu. m.

$= 20373.7310$ cu. m.

Let the radius be 'r' m. of each of the small spheres newly made.

$\therefore \ 3 \times \frac{4}{3} \pi r^3 = 20373.7310$

or, $r^3 = \frac{20373.7310}{4\pi}$

$= 1621.29$

$r = 11.7477$ m.

If a circle having the radius 11.75 sq. is drawn on the graph paper then it will be the proportional diagram of the new sphere.

![Diagram-27](image-url)
CHAPTER SIX
VECTOR

Problem
Draw three vectors \( \mathbf{a}, \mathbf{b}, \mathbf{c} \) so that they are not parallel to one another and then draw the vector \( \mathbf{a} - 3 \mathbf{b} + 2 \mathbf{c} \).

Theory
If a vector \( \mathbf{v} \) be drawn from the terminal point of another vector \( \mathbf{u} \), then \( \mathbf{u} + \mathbf{v} \) is meant such a vector whose initial point is that of \( \mathbf{u} \) and whose terminal point is that of \( \mathbf{v} \).

Procedure
1. First we draw three vectors of any length in any direction which is not parallel to one another. Let the length of the vectors \( \mathbf{a}, \mathbf{b}, \mathbf{c} \) be 2, 2.5 and 4 cm. respectively.

2. From any point \( P \) draw a vector \( \overrightarrow{PQ} \) equal and parallel to \( \mathbf{a} \). From the end point \( Q \) of the vector \( \mathbf{a} \) draw vector \( \overrightarrow{QR} = -3 \mathbf{b} \) in opposite direction of the vector \( \mathbf{b} \).

3. According to the triangle law of vector addition, \( \mathbf{a} + (-3 \mathbf{b}) = \overrightarrow{PR} \).

4. We draw the vector \( \overrightarrow{RS} = 2 \mathbf{c} \) from the point \( R \) parallel to \( \mathbf{c} \). Now joining the term point \( S \) of \( 2 \mathbf{c} \) with the initial point \( P \) of the vector \( \mathbf{a} \), we get the vector \( \overrightarrow{PS} \).

5. By the triangle law of vector addition, we get
\[
\overrightarrow{PS} = \overrightarrow{PR} + \overrightarrow{RS},
\]
that is,
\[
\overrightarrow{PS} = \mathbf{a} - 3 \mathbf{b} + 2 \mathbf{c}.
\]
Problem No. 6.2

Date:

**Problem**
From a certain point an aeroplane moves towards west at a distance of 200 km. then making an angle of 60° on its way it moves 150 km. to the same direction. Determine the distance of the aeroplane from it's starting point to the destination. Justify the construction geometrically.

**Theory**
If a vector \( \mathbf{v} \) be drawn from the terminal point of another vector \( \mathbf{u} \), then \( \mathbf{u} + \mathbf{v} \) is meant such as vector whose initial point is that of \( \mathbf{u} \) and whose terminal point is that of \( \mathbf{v} \).

**Procedure**
1. Take a piece of graph paper and let the scale be 1 sq. = 10 km. which will be treated as the scale of construction.
2. From any point A, draw the line AB = 20 sq. to the left (i.e. the west)
3. Draw angle \( \angle ABC = 120^\circ \) at the point B with BA, the direction of the aeroplane and take BC = 15 sq. The line BC in not parallel to any line of
the graph. So to find the actual position of the point, let us draw an angle \( \angle A \) \( \text{BC} = 60 \). Draw an arc considering the centre at B and radius 15 sq. (as per scale). The arc intersects BC at the point C.

4. Join CA. Then CA is the required distance.

5. We cut AC \( \cong AC \) from AA, we see from the graph paper that AC \( \cong 30 \) sq. unit (Approximately).

\[ AC = 300 \text{ km. (Approximately).} \]

**Verification**

In the diagram AN = 27 unit

\[
\begin{align*}
\text{CN} & = 13 \text{ unit.} \\
\text{AC} & = \sqrt{AN^2 + CN^2} \text{ unit} \\
& = \sqrt{27^2 + 13^2} \text{ unit} \\
& = 29.9666 \text{ unit} \\
& = 299.7 \text{ km.}
\end{align*}
\]

By practical measurement,

\[ AC = 300 \text{ km. (approx).} \]